Kuwait University — Math.102 — 15 October, 1998

Dept.of Math.&Comp.Sci. — First Midterm — Duration: 75 min

Answer all the following questions. Calculators and mobile phones are *not* allowed. Each (sub)question carries 5 marks.

1. (a) Show that the inverse of $f(x) = \sin^{-1}(\ln x) + \pi/2$ exists for $1/e < x < e$, and find $f^{-1}(x)$. State the domain and range of f^{-1}

   (b) Let $h(x) = 2x^3 + 3^{2x}$. Find an equation for the tangent to the graph of $y = h^{-1}(x)$ at the point $(1, 0)$.

2. Find $\frac{dy}{dx}$ if

   (a) $y = \ln \tan^3(3x) + \cos \ln(2x)$

   (b) $y = \sqrt[5]{e^{3x^3} \sin x}$;

   (c) $y = (\log_5(\ln x^2))^{\tan x}$.

3. Evaluate

   (a) $\int (1 + \csc x)^2 \, dx$;

   (b) $\int \frac{\cot(e^{-3x})}{e^{3x}} \, dx$;

   (c) $\int \frac{\sinh x}{5 + \sinh^2 x} \, dx$.

4. Find the exact value of $\sec(\tan^{-1}(\frac{4}{3})) + \tanh(\ln 2)$.

5. Find $\lim_{x \to 0} \frac{\sin^{-1}(1 - \cos x)}{x^2}$.